

## Constants and units

$$R = 8.3145 \text{ J K}^{-1} \text{ mol}^{-1} \\ = 0.08206 \text{ L atm mol}^{-1} \text{ K}^{-1}$$

$$k_B = 1.381 \times 10^{-23} \text{ J K}^{-1} \\ = 1.381 \times 10^{-16} \text{ g cm}^2 \text{ s}^{-2} \text{ K}^{-1}$$

$$N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$$

$$c = 2.998 \times 10^8 \text{ m s}^{-1}$$

$$h = 6.626 \times 10^{-34} \text{ J s}$$

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34} \text{ J s}$$

$$1 \text{ N} = 1 \text{ kg m s}^{-2}$$

$$1 \text{ J} = 1 \text{ N m} = 1 \text{ kg m}^2 \text{ s}^{-2}$$

$$1000 \text{ L} = 1 \text{ m}^3$$

$$\ln N! \approx N \ln N - N$$

$$\ln(M - N) \approx \ln M \text{ if } M \gg N$$

## Quantum mechanics

$$E = h\nu = \frac{hc}{\lambda}$$

$$E_{1D \text{ PIB}} = \frac{n^2 h^2}{8mL^2}, n = 1, 2, 3, \dots$$

$$E_{RR} = \frac{\hbar^2 J(J+1)}{2I}, J = 0, 1, 2, \dots$$

$$E_{HO} = \hbar \left( \frac{k}{\mu} \right)^{1/2} \left( v + \frac{1}{2} \right), v = 0, 1, 2, \dots$$

## Molecular transport

$$J(x) = -D \left[ \frac{\partial c(x)}{\partial x} \right]$$

$$\frac{\partial c(x,t)}{\partial t} = D \frac{\partial^2 c(x,t)}{\partial x^2}$$

$$D = \frac{k_B T}{f}$$

$$f_{\text{sphere}} = 6\pi\eta r$$

$$f_{\text{disk, random}} = 12\eta r$$

$$\langle r^2 \rangle_{3D}^{1/2} = \sqrt{6Dt}$$

## Kinetic molecular theory, gas collisions

$$\sigma = \pi d^2; z_{AA} = \sqrt{2}\rho\sigma \langle v \rangle; t = \frac{1}{z_{AA}}$$

$$l = \frac{\langle v \rangle}{z} = \frac{1}{\sqrt{2}\rho\sigma}$$

$$Z_{AA} = \frac{\sigma \langle v \rangle \rho^2}{\sqrt{2}}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

## Maxwell-Boltzmann speed distribution

$$P(v) = 4\pi \left( \frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-Mv^2/2RT}$$

$$\langle v \rangle = \left( \frac{8RT}{\pi M} \right)^{1/2}$$

$$\langle v^2 \rangle^{1/2} = \left( \frac{3RT}{M} \right)^{1/2}$$

$$\langle v_{mp} \rangle = \left( \frac{2RT}{M} \right)^{1/2}$$

## Kinetics

$$k = Ae^{-\frac{E_a}{RT}}$$

$$t_{1/2} = \frac{\ln 2}{k} \text{ first-order only}$$

$$v(t) = -\frac{1}{a} \frac{d[A]}{dt} = \dots \\ = -\frac{1}{V} \frac{d\xi}{dt}$$

$$K_c = \frac{k_1}{k_{-1}} = \frac{[B]_{eq}}{[A]_{eq}}$$

$$\Delta[B] = \Delta[B]_0 e^{-t/\tau}; \tau = \frac{1}{k_1 + k_{-1}}$$

$$k = \frac{k_B T}{hc^\circ} e^{\frac{\Delta^\ddagger S^\circ}{R}} e^{-\frac{\Delta^\ddagger H^\circ}{RT}} \\ = \frac{k_B T}{hc^\circ} e^2 \cdot e^{\frac{\Delta^\ddagger S^\circ}{R}} e^{-\frac{E_a}{RT}}$$

$$E_a = \Delta^\ddagger H^\circ + 2RT$$

$$\text{yield}_i = \frac{k_i}{\sum_n k_n}$$

$$\frac{d[I]}{dt}_{ss} = 0; [I]_{ss} = \frac{k_1}{k_2} [A]_0 e^{-k_1 t}$$

$$[ES]_{ss} = \frac{k_1 [E] [S]}{k_{-1} + k_2} \\ = \frac{[E]_0 [S]}{K_M + [S]}$$

$$K_M = \frac{k_{-1} + k_2}{k_1}$$

$$\frac{d[P]}{dt} = k_2 [ES]_{ss} \\ = k_2 [E]_0 \frac{[S]}{K_M + [S]} \\ = v_{max} \frac{[S]}{K_M + [S]}$$

$$\frac{1}{d[P]/dt} = \frac{1}{v_{max}} + \frac{K_M}{v_{max} [S]}$$

$$v = \frac{v_{max} [S]}{K_M + [S]}$$

## Chemical and phase equilibria

$$\mu = \left( \frac{\partial G}{\partial n} \right)_{p,T}$$

$$\frac{dp}{dT} = \frac{\Delta S}{\Delta V} = \frac{\Delta H}{T\Delta V}$$

$$\ln \frac{p_2}{p_1} = -\frac{\Delta_{vap}H}{R} \left( \frac{1}{T_2} - \frac{1}{T_1} \right)$$

$$\ln \frac{K_p(T_2)}{K_p(T_1)} = -\frac{\Delta_{rxn}H}{R} \left( \frac{1}{T_2} - \frac{1}{T_1} \right)$$

$$\Delta_{rxn}G^\circ(T) = -RT \ln K_p(T)$$

$$\Delta_{rxn}G = \Delta_{rxn}G^\circ + RT \ln Q$$

$$Q = \frac{p_Y^{v_Y} p_Z^{v_Z} \dots}{p_A^{v_A} p_B^{v_B} \dots}$$

$$K_p = \left( \frac{p_Y^{v_Y} p_Z^{v_Z} \dots}{p_A^{v_A} p_B^{v_B} \dots} \right)_{eq}$$

$$K_c = \left( \frac{c_Y^{v_Y} c_Z^{v_Z} \dots}{c_A^{v_A} c_B^{v_B} \dots} \right)_{eq}$$

$$= \frac{\left(\frac{q_Y}{V}\right)^{v_Y} \left(\frac{q_Z}{V}\right)^{v_Z} \dots}{\left(\frac{q_A}{V}\right)^{v_A} \left(\frac{q_B}{V}\right)^{v_B} \dots}$$

$$K_p = K_c \left( \frac{c^\circ}{p^\circ} RT \right)^{v_Y + v_Z + \dots - v_A - v_B - \dots}$$

$$\frac{q}{V} = \frac{q_{trans}}{V} q_{rot} q_{vib} q_{elec}$$

$$= \frac{q^\circ}{V} e^{D_0/k_B T}$$

$$q_{vib} q_{elec} = \frac{1}{1 - e^{-\theta/T}} \cdot g \cdot e^{D_0/RT}$$

$$\frac{q^\circ}{V} = \frac{N_A p^\circ}{RT} e^{-(G^\circ - H_0^\circ)/RT}$$

## Thermodynamics

$$\delta w = -p_{ext} dV; w = -p_{ext} \Delta V$$

$$w_{iso-T} = -\int_{V_1}^{V_2} \frac{nRT}{V} dV = -nRT \ln \frac{V_2}{V_1}$$

$$C_V = \left( \frac{\partial U}{\partial T} \right)_V; C_p = \left( \frac{\partial H}{\partial T} \right)_p$$

$$\Delta H = \int dH = \int C_p(T) dT$$

$$C_p = C_V + nR$$

$$\Delta U = w + q$$

$$\Delta S = \frac{q_{rev}}{T} = nR \ln \frac{V_2}{V_1} + C_V \ln \frac{T_2}{T_1}$$

$$\Delta S = \int \frac{C_p}{T} dT + \sum_i \frac{\Delta_{trs}H}{T}$$

$$\left[ \frac{S}{R} = \frac{7}{2} + \ln \left[ \left( \frac{2\pi m k_B T}{h^2} \right)^{3/2} \frac{V}{N} \right] + \ln \left[ \frac{T}{\sigma \theta_{rot}} \right] \dots \right.$$

$$\left. \dots + \frac{\theta_{vib}}{T} \left( \frac{1}{e^{\theta_{vib}/T} - 1} \right) - \ln \left[ 1 - e^{-\theta_{vib}/T} \right] + \ln [g_{el}] \right]$$

$$\frac{p_2}{p_1} = \left( \frac{V_1}{V_2} \right)^\gamma$$

$$V_2 = \frac{V_1}{\gamma} \left[ \gamma - 1 + \frac{p_1}{p_2} \right]$$

$$\gamma = C_p / C_V$$

$$H = U + pV; A = U - TS$$

$$G = U - TS + pV = H - TS = A + pV$$

$$dU = TdS - pdV; \left( \frac{\partial T}{\partial V} \right)_S = - \left( \frac{\partial p}{\partial S} \right)_V$$

$$dH = TdS + Vdp; \left( \frac{\partial T}{\partial p} \right)_S = \left( \frac{\partial V}{\partial S} \right)_p$$

$$dA = -SdT - pdV; \left( \frac{\partial S}{\partial V} \right)_T = \left( \frac{\partial p}{\partial T} \right)_V$$

$$dG = -SdT + Vdp; \left( \frac{\partial S}{\partial p} \right)_T = - \left( \frac{\partial V}{\partial T} \right)_p$$

$$\left( \frac{\partial (\Delta G/T)}{\partial T} \right)_p = -\frac{\Delta H}{T^2}$$

$$\mu_{JT} = \left( \frac{\partial T}{\partial p} \right)_H = \frac{T \left( \frac{\partial V}{\partial T} \right)_p - V}{C_p}$$

## Statistical mechanics

$$\frac{a_n}{a_m} = e^{-\beta \Delta E}; \beta = \frac{1}{k_B T}$$

$$p_j = \frac{e^{-\beta E_j}}{\sum_i e^{-\beta E_i}} = \frac{e^{-\beta E_j}}{Q}$$

$$Q(N, V, \beta) = \sum_i e^{-\beta E_i} = \frac{q^N}{N!}$$

$$q_{trans} = \left( \frac{2\pi m k_B T}{h^2} \right)^{3/2} V$$

$$q_{rot} \approx \frac{T}{\sigma \theta_{ROT}}; \theta_{ROT} = \frac{\hbar^2}{2Ik_B}, I = \mu r^2$$

$$q_{vib} = \frac{e^{-\theta_{VIB}/2T}}{1 - e^{-\theta_{VIB}/T}}$$

$$\theta_{VIB} = \frac{\hbar \left( \frac{k}{\mu} \right)^{1/2}}{k_B} = \frac{h\nu}{k_B}$$

$$\langle E \rangle = U = - \left( \frac{\partial \ln Q}{\partial \beta} \right)_{N,V}$$

$$= k_B T^2 \left( \frac{\partial \ln Q}{\partial T} \right)_{N,V}$$

$$\langle p \rangle = \frac{1}{Q\beta} \left( \frac{\partial Q}{\partial V} \right)_{N,T} = k_B T \left( \frac{\partial \ln Q}{\partial V} \right)_{N,T}$$

$$S = k_B \ln W = -k_B \sum_i p_i \ln p_i$$

$$S = k_B T \left( \frac{\partial \ln Q}{\partial T} \right)_{N,V} + k_B \ln Q$$

$$H = k_B T \left[ T \left( \frac{\partial \ln Q}{\partial T} \right)_{N,V} + V \left( \frac{\partial \ln Q}{\partial V} \right)_{N,T} \right]$$

$$A = -k_B T \ln Q$$

$$G = k_B T \left[ V \left( \frac{\partial \ln Q}{\partial V} \right)_{N,T} - \ln Q \right]$$

$$\mu = -RT \left( \frac{\partial \ln Q}{\partial N} \right)_{V,T}$$

$$W(M, N) = \frac{M!}{N!(M-N)!}$$